



Conserved Charge Fluctuations from Lattice QCD and the Beam Energy Scan

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Abstract

We discuss the next-to-leading order Taylor expansion of ratios of cumulants of net-baryon number fluctuations. We focus on the relation between the skewness ratio, $S_B\sigma_B \equiv \chi_3^B/\chi_1^B$, and the kurtosis ratio, $\kappa_B\sigma_B^2 \equiv \chi_4^B/\chi_2^B$. We show that differences in these two cumulant ratios are small for small values of the baryon chemical potential. The next-to-leading order correction to $\kappa_B\sigma_B^2$ however is approximately three times larger than that for $S_B\sigma_B$. The former thus drops much more rapidly with increasing beam energy, $\sqrt{s_{NN}}$. We argue that these generic patterns are consistent with current data on cumulants of net-proton number fluctuations measured by the STAR Collaboration at $\sqrt{s_{NN}} \geq 19.6$ GeV.

Keywords: Quark-Gluon Plasma, Lattice QCD, Heavy Ion Collisions

1. Introduction

Fluctuations of conserved charges of strong interactions have long been considered as a set of sensitive observables to explore the structure of the phase diagram of Quantum Chromodynamics (QCD). They currently are the most promising experimental observables in the search for a possible critical point in the phase diagram of QCD that gets performed with the beam energy scan (BES) program at the Relativistic Heavy Ion Collider (RHIC). Although the results on net-electric charge [1, 2] and net-proton number [3, 4] fluctuations obtained from the first BES runs performed at RHIC did not yet provide clear cut evidence for the existence of a critical point, the collected data on charge fluctuations show an intriguing dependence on the beam energy which at present is poorly understood even qualitatively. The published data on cumulants of net-proton number fluctuations [3] and, in particular, the still preliminary data set, which covers a larger transverse momentum range [4], show obvious deviations from the thermodynamics of a hadron resonance

gas (HRG), e.g. the ratios of even as well as odd order cumulants differ from unity and different mixed ratios, formed from even and odd order cumulants, are not identical. This may not be too surprising as HRG model calculations are not expected to give an accurate description of higher order cumulants close to the QCD (phase) transition region. It, however, raises the question whether the observed pattern seen in measured net-proton fluctuations can be understood in terms of QCD thermodynamics or whether other effects are responsible for this.

We will discuss net-proton number fluctuations at small values of the baryon chemical potential, μ_B . In particular, we will point out basic features observed in ratios of cumulants of net-proton number fluctuations in the BES at RHIC, which can not be accommodated in HRG model calculations, but are naturally explained in QCD using a next-to-leading (NLO) order expansion of cumulant ratios in terms of μ_B .

2. Cumulants of net-baryon number fluctuations

We will discuss here the structure of ratios of net-baryon number cumulants that can be formed with the first four cumulants which are related to mean (M_B), variance (σ_B^2), skewness (S_B) and kurtosis (κ_B) of net baryon number distributions,

$$\begin{aligned} R_{12}^B(T, \mu_B, \mu_Q, \mu_S) &\equiv \frac{\chi_1^B(T, \mu_B, \mu_Q, \mu_S)}{\chi_2^B(T, \mu_B, \mu_Q, \mu_S)} \equiv \frac{M_B}{\sigma_B^2}, \quad R_{31}^B(T, \mu_B, \mu_Q, \mu_S) \equiv \frac{\chi_3^B(T, \mu_B, \mu_Q, \mu_S)}{\chi_1^B(T, \mu_B, \mu_Q, \mu_S)} \equiv \frac{S_B \sigma_B^3}{M_B}, \\ R_{42}^B(T, \mu_B, \mu_Q, \mu_S) &\equiv \frac{\chi_4^B(T, \mu_B, \mu_Q, \mu_S)}{\chi_2^B(T, \mu_B, \mu_Q, \mu_S)} \equiv \kappa_B \sigma_B^2. \end{aligned} \quad (1)$$

Here the n -th order cumulants are obtained as partial derivatives of the QCD pressure, $P(T, \mu_B, \mu_Q, \mu_S)$, with respect to the chemical potentials,

$$\chi_n^B(T, \mu_B, \mu_Q, \mu_S) = \frac{\partial^n P / T^4}{\partial (\mu_B / T)^n} \quad \text{or} \quad \chi_{nm}^{BX}(T, \mu_B, \mu_Q, \mu_S) = \frac{\partial^{(n+m)} P / T^4}{\partial (\mu_B / T)^n \partial (\mu_X / T)^m}, \quad X = Q, S. \quad (2)$$

Using Eq. 1 we also obtain $R_{32}^B \equiv S_B \sigma_B$, as $R_{32}^B = R_{31}^B R_{12}^B$. For this ratio and those introduced in Eq. 1 we may set up Taylor expansions in terms of the chemical potentials. Here it is convenient to introduce constraints that resemble thermal conditions met in heavy ion collisions, i.e. we demand strangeness neutrality $M_S \equiv \chi_1^S(T, \mu_B, \mu_Q, \mu_S) = 0$ and fix a relation between net-electric charge (M_Q) and net baryon number (M_B). We may choose $M_Q = r M_B$ with $r = 0.4$, which resembles electric charge to baryon number ratio in the incident beams in heavy ion collisions at RHIC and LHC. With these conditions the chemical potentials μ_Q and μ_S become functions of μ_B , i.e. to leading order one has $\mu_S / \mu_B = s_1(r)$, $\mu_Q / \mu_B = q_1(r)$ [5]. The ratios R_{nm}^B introduced in Eq. 1 then become functions of T and μ_B only. They may then be Taylor expanded in μ_B ,

$$R_{nm}^B(T, \mu_B) = R_{nm}^{B,0}(T) + R_{nm}^{B,2}(T) \left(\frac{\mu_B}{T} \right)^2 + O(\mu_B^4). \quad (3)$$

We will focus here on the relation between the expansion coefficients for the skewness ratio $R_{31}^B \equiv S_B \sigma_B^3 / M_B$ and those of the kurtosis ratio $R_{42}^B \equiv \kappa_B \sigma_B^2$. In a medium with vanishing strangeness and electric charge chemical potential this relation is, in fact, quite simple:

$$\mu_Q = \mu_S = 0 : \quad R_{42}^{B,0}(T) = R_{31}^{B,0}(T), \quad R_{42}^{B,2}(T) = 3 R_{31}^{B,2}(T) \quad (4)$$

However, when implementing the constraints $M_S = 0$ and $M_Q / M_B = r$, such simple relations no longer hold. In fact, in this case the leading order (LO) coefficients are related through

$$\underline{M_S = 0, \quad M_Q / M_B = r :} \quad R_{42}^{B,0}(T) - R_{31}^{B,0}(T) = \frac{s_1(\chi_{31}^{BS} \chi_2^B - \chi_{11}^{BS} \chi_4^B) + q_1(\chi_{31}^{BQ} \chi_2^B - \chi_{11}^{BQ} \chi_4^B)}{(\chi_2^B + s_1 \chi_{11}^{BS} + q_1 \chi_{11}^{BQ}) \chi_2^B}, \quad (5)$$

The corresponding expressions for the NLO corrections are somewhat more involved. They will be presented elsewhere [6].

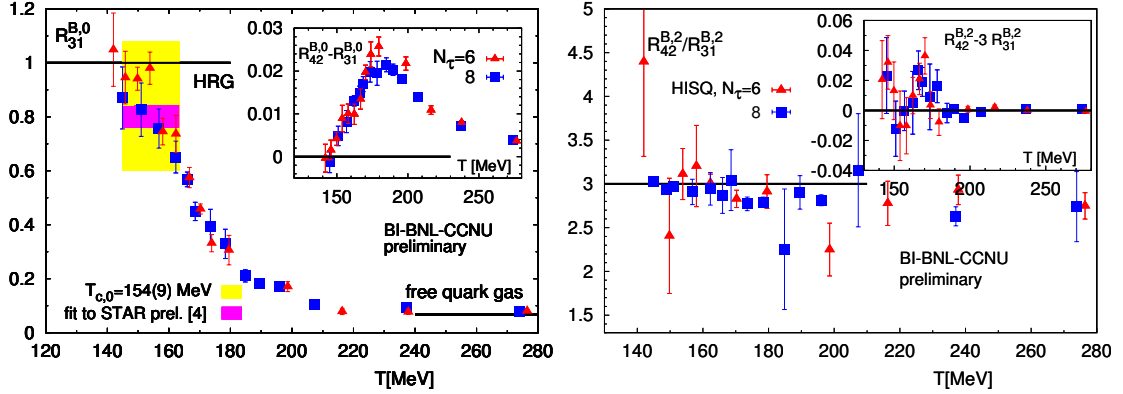


Fig. 1. Left: The leading order ($R_{31}^{B,0}$) result for the skewness ratio R_{31}^B versus temperature calculated in (2+1)-flavor QCD on lattices of size $(4N_\tau)^3 \cdot N_\tau$ with $N_\tau = 6, 8$ for a strangeness neutral system with $r = 0.4$. The insertion shows the difference between the LO expansion coefficients for R_{42}^B and R_{31}^B . Right: The ratio of the NLO expansion coefficients, $R_{42}^{B,2}$ and $R_{31}^{B,2}$. The insertion shows the corresponding difference $R_{42}^{B,2} - 3R_{31}^{B,2}$. Horizontal lines in the right hand figure corresponds to the result for $\mu_S = \mu_Q = 0$. For details see discussion in text.

In Fig. 1(left) we show results for $R_{31}^{B,0}$, i.e. the leading order result for R_{31}^B at $\mu_B = 0$. The insertion in this figure shows the difference $R_{42}^{B,0}(T) - R_{31}^{B,0}(T)$. As can be seen, for temperatures in the crossover region, $T_{c,0} = (154 \pm 9)$ MeV (yellow band in main panel), the magnitude of this difference is at most 2% of $R_{31}^{B,0}(T)$ but may reach about 10% at $T \approx 180$ MeV. This suggests that the skewness ratio $R_{31}^B = S_B \sigma_B^3 / M_B$ and the kurtosis ratios $R_{42}^B = \kappa_B \sigma_B^2$ should be almost identical at the highest RHIC energies, where $\mu_B / T \approx 0.15$.

In Fig. 1(right) we show the ratio of the NLO expansion coefficients $R_{42}^{B,2}(T)$ and $R_{31}^{B,2}(T)$. For $\mu_Q = \mu_S = 0$ it is straightforward to show that this ratio equals three as stated in Eq. 4. In the constraint case this, however, does not need to be the case. In fact, in the infinite temperature limit the ratio varies between 5/3 and 2, depending on the value of $M_Q / M_B = r$. Precise calculations of the ratio $R_{42}^{B,2} / R_{31}^{B,2}$ are demanding as one needs to evaluate 6th order cumulants and both expansion coefficients may change sign in the temperature range of interest. With the presently available statistics this causes the large errors on $R_{42}^{B,2} / R_{31}^{B,2}$ seen in Fig. 1(right) for some values of T . It is, however, evident that the ratio of NLO expansion coefficients stays close to 3 in a wide T -range. Hence one may expect that the dependence of the kurtosis ratio, $\kappa_B \sigma_B^2 = \chi_4^B / \chi_2^B$, on μ_B , and thus on $\sqrt{s_{NN}}$, is significantly larger than that of the skewness ratio $S_B \sigma_B^3 / M_B = \chi_3^B / \chi_1^B$.

3. Cumulants of net-proton number fluctuations

We compare these generic features of the relation of LO and NLO expansion coefficients of the Taylor expansion of cumulant ratios R_{42}^B and R_{31}^B with experimental data on ratios of cumulants of net proton number fluctuations measured in the BES at RHIC. To do so, we also note that we may eliminate the dependence of these cumulant ratios on μ_B , by solving the Taylor series expansion for another ratio, e.g. $R_{12}^B(T, \mu_B) = M_B / \sigma_B^2 \equiv R_{12}^{B,1}(T) \mu_B / T + O(\mu_B^3)$. Eliminating μ_B / T in Taylor expansions in favor of R_{12}^B is possible as long as the relation between both is unique. This will not be the case in general, but seems to hold in the (T, μ_B) regime covered in the BES. As can be seen from Fig. 2(left), along the freeze-out line the ratio R_{12}^P is a monotonically decreasing function of the beam energy, i.e. R_{12}^P rises with increasing μ_B .

In Eq. 3 we thus may replace μ_B / T by $(R_{12}^{B,1})^{-1} M_B / \sigma_B^2$. Experimentally one cannot directly measure net-baryon number fluctuations and their cumulants. One rather has access only to cumulants of net-proton number fluctuations. It then is appropriate to consider the ratios R_{42}^P and R_{31}^P as functions of R_{12}^P and test to what extent the generic features discussed for the corresponding ratios R_{42}^B and R_{31}^B are reflected in the data. In Fig. 2 we show preliminary data on ratios of various net-proton number cumulants obtained by the STAR Collaboration in the transverse momentum range $0.4 \text{ GeV} \leq p_t \leq 2.0 \text{ GeV}$. The left hand figure shows

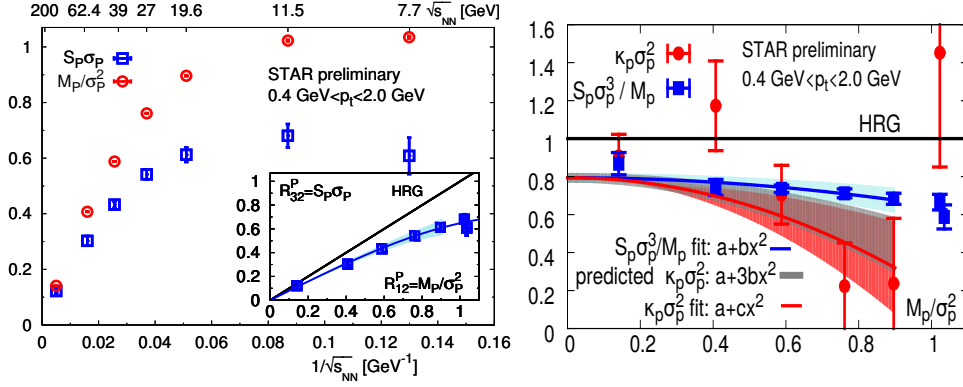


Fig. 2. Left: Preliminary data for $R_{12}^P = M_P/\sigma_P^2$ and $R_{32}^P = S_P\sigma_P$ obtained by the STAR Collaboration in the transverse momentum interval, $0.5 \text{ GeV} \leq p_t \leq 2.0 \text{ GeV}$. Right: Results for $R_{31}^P = \chi_3^P/\chi_1^P$ constructed from the data shown in the left hand part of the figure and preliminary data for $R_{42}^P = \chi_4^P/\chi_2^P$ (the data point for R_{42}^P at $\sqrt{s_{NN}} = 7.7 \text{ GeV}$ is not shown). For details see discussion in the text.

that $S_P\sigma_P < M_P/\sigma_P^2$ holds in the entire energy range covered in the BES at RHIC. This is clearly different from HRG model predictions, where these two cumulant ratios are identical. Such a difference, however, naturally arises in QCD thermodynamics. As can be seen in Fig. 1(left) the ratio $R_{31}^B = R_{32}^B/R_{12}^B$ is smaller than unity for all T . A quadratic fit to the STAR data for R_{31}^P vs. R_{12}^P , for $R_{12}^P \leq 0.9$, or equivalently for beam energies $\sqrt{s_{NN}} \geq 19.6 \text{ GeV}$, yields $R_{31}^P = 0.80(4) - 0.15(5)(R_{12}^P)^2$. The intercept at $R_{12}^P = 0$, i.e. at $\mu_B = 0$, is shown in Fig. 1(left) as a horizontal bar. It is consistent with a freeze-out temperature at or below the QCD transition temperature, $T_{c,0}$. This also is consistent with freeze-out temperatures obtained from an analysis of cumulants of net-electric charge fluctuations as functions of R_{12}^P [7].

Fig. 2(right) shows the STAR data for R_{31}^P and R_{42}^P . The latter have large statistical and systematic errors. The grey band shown in this figure is the expected behavior of R_{42}^P when using knowledge on R_{31}^P as input and assuming that the data on net-proton number fluctuations follow the generic behavior discussed above for net-baryon number fluctuations in QCD at small values of μ_B , i.e. it shows three times the slope obtained from a fit to R_{31}^P (light blue band). Performing a fit to the data for R_{42}^P that is constrained at $\mu_B = 0$ by assuming $R_{42}^P(\mu_B = 0) = R_{31}^P(\mu_B = 0)$ yields, $R_{42}^P = 0.80 - 0.59(30)(R_{12}^P)^2$. This is shown by the light red band. Although errors on the data are still large, this result is consistent with an expected factor three larger curvature coefficient for the data on R_{42}^P with respect to the data on R_{31}^P .

4. Conclusions

Data on cumulants of higher order net-proton number fluctuations taken at RHIC at beam energies $\sqrt{s_{NN}} \geq 19.6 \text{ GeV}$ are consistent with expectations deduced from QCD calculations for cumulants of net-baryon number fluctuations performed in a NLO Taylor expansion in μ_B . In particular, the strong decrease of $\kappa_P\sigma_P^2$ relative to the mild variation of $S_P\sigma_P$ is consistent with "non-critical" behavior of cumulant ratios.

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